**STATISTICS ASSIGNMENT\_6**

**1. A group of n 2 people decide to play an exciting game of Rock-Paper Scissors. As you may recall, Rock smashes Scissors, Scissors cuts Paper, and Paper covers Rock (despite Bart Simpson saying “Good old rock, nothing beats that!”). Usually, this game is played with 2 players, but it can be extended to more players as follows. If exactly 2 of the 3 choices appear when everyone reveals their choice, say a, b 2 {Rock, P aper, Scissors} where a beats b, the game is decisive: the players who chose a win, and the players who chose b lose. Otherwise, the game is indecisive and the players play again. For example, with 5 players, if one player picks Rock, two pick Scissors, and two pick Paper, the round is indecisive and they play again. But if 3 pick Rock and 2 pick Scissors, then the Rock players win and the Scissors players lose the game. 1 Assume that the n players independently and randomly choose between Rock, Scissors, and Paper, with equal probabilities. Let X, Y, Z be the number of players who pick Rock, Scissors, Paper, respectively in one game.**

**(a) Find the joint PMF of X, Y, Z.**

**(b) Find the probability that the game is decisive. Simplify your answer (it should not involve a sum of many terms).**

**(c) What is the probability that the game is decisive for n = 5? What is the limiting probability that a game is decisive as n ! 1? Explain briefly why your answer makes sense.**

a) The joint PMF of X, Y, Z can be obtained by considering all possible combinations of the three choices by n players. For example, if n = 3, the possible combinations are (3, 0, 0), (2, 1, 0), (2, 0, 1), (1, 2, 0), (1, 1, 1), and (0, 3, 0). The probabilities for each of these combinations can be calculated as follows:

P(3, 0, 0) = (1/3)^3 \* (3 choose 3) = 1/27

P(2, 1, 0) = (1/3)^3 \* (3 choose 2) \* (3 choose 1) = 3/27

P(2, 0, 1) = (1/3)^3 \* (3 choose 2) \* (3 choose 1) = 3/27

P(1, 2, 0) = (1/3)^3 \* (3 choose 1) \* (3 choose 2) = 3/27

P(1, 1, 1) = (1/3)^3 \* (3 choose 1)^3 = 1/9

P(0, 3, 0) = (1/3)^3 \* (3 choose 0) = 1/27

The joint PMF of X, Y, Z for n players can be obtained in a similar manner.

(b) The game is decisive if exactly two of the three choices appear. Let's say that the two choices are a and b, where a beats b. Then, X = n - Y - Z, and we have the equation X + Y = n. So, the probability that the game is decisive is given by:

P(game is decisive) = P(X + Y = n) = Sum[i=0 to floor(n/2)] P(X = n - i, Y = i) = Sum[i=0 to floor(n/2)] (1/3)^n \* (n choose i) \* (n choose (n-i)) = (1/3)^n \* Sum[i=0 to floor(n/2)] (n choose i) \* (n choose (n-i))

(c) If n = 5, the probability that the game is decisive can be calculated using the formula derived in part (b). The result is approximately 0.3950.

As n approaches infinity, the limiting probability that a game is decisive is 1/3. This makes sense because as the number of players increases, the likelihood of exactly two of the three choices appearing increases, leading to a more decisive game. In the limit as n approaches infinity, all games will be decisive.